

Hawking radiation in a waveguide is produced by self-phase modulation.

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Recently it was suggested that the Hawking radiation may be observed in an electromagnetic waveguide (PRL 95, 031301 (2005)). We show that the Hawking effect in a waveguide is identical to the well-known effect of frequency broadening of an optical pulse due to self-phase modulation.

In 1974 Hawking showed that black holes can evaporate by emission of thermal radiation¹. A closely related effect had been introduced a few years later by Unruh. He showed that for an accelerating observer vacuum should look like a bath of thermal radiation with temperature T_{UH} defined as

$$k_B T_{UH} = \frac{\hbar a}{2\pi c}, \quad (1)$$

where a is the acceleration of the observer². The Hawking temperature may be obtained from this formula by substitution of the free fall acceleration near the black hole horizon into eq.(1). While well-established theoretically, these effects are believed to be very difficult to observe in the experiment: an observer accelerating at $a = g = 9.8m/s^2$ should see vacuum temperature of only $4 \times 10^{-20}K$. Over the past years quite a few proposals were introduced on how to emulate and/or observe the Unruh-Hawking effect. A recent review of these proposals, which involves various acoustic, optical, accelerator physics, and solid state physics situations can be found in³. Very recently it was suggested that the Hawking radiation may potentially be observed in optical waveguides⁴. In other recent proposals it was suggested that the Unruh-Hawking radiation may have already been detected in the experiments with surface plasmon polaritons⁵ which propagate over a curved metal surface⁶, and that the Unruh effect may be emulated in tapered optical waveguides⁷. However, despite large number of different proposals, no one has reported yet an undisputed experimental observation of the Hawking radiation.

In this paper we demonstrate that in the recently proposed model of the Hawking effect in an electromagnetic waveguide⁴ the Hawking radiation may be interpreted as the well-known effect of frequency broadening of an optical pulse due to self-phase modulation. This effect is well established in fiber optics both theoretically and experimentally. Thus, there is no need for further experimental verification of the Hawking radiation.

Following ref.⁴ and the closely related ref.⁸ let us consider an initially monochromatic short optical pulse of frequency ω_0 , which propagates along an optical fiber (Fig.1). At large enough intensity I of the pulse the refractive index of the fiber n is modified by the propagating pulse due to the Kerr nonlinearity:

$$n = n_0 + n_2 I \quad (2)$$

For the sake of simplicity let us assume that the n_2 coefficient does not exhibit any dependence on the frequency. On the other hand, let us assume that the waveguide exhibits the so-called anomalous dispersion: $dn_0/d\omega < 0$ in the wide enough frequency range around ω_0 . This means that at low frequencies ($\omega < \omega_0$) the speed of photons in the waveguide is smaller than the speed of the propagating optical pulse. In principle, $n_2 > 0$ and $dn_0/d\omega < 0$ corresponds to the conditions for the formation of an optical soliton in the fiber. However, for the sake of simplicity let us not be concerned about the stability of the propagating optical pulse.

The situation shown in Fig.1 in the reference frame commoving with the optical pulse replicates the picture of two horizons for the low-frequency photons, which is similar to the one shown in Fig.2 of ref.⁸: the front edge of the optical pulse behaves as a black hole horizon for low-frequency photons in the fiber, while the trailing edge of the pulse behaves as a white hole horizon. The low frequency photons (if any) in front of the front edge of the pulse cannot escape it, because their speed is smaller than the speed of the pulse. On the other hand, the low frequency photons behind the trailing edge of the pulse cannot reach the trailing edge. This qualitative picture of the toy black hole and white hole horizons in an optical fiber is intuitively clear. The mathematical justification of this picture in terms of the effective metric, effective surface gravity, etc. which is experienced by the low frequency photons can be found in refs.^{4,8}. The resulting expression for the temperature of the Hawking radiation may be found in refs.^{4,7,8,9}:

$$k_B T_{UH} = \frac{\hbar}{2\pi} \left| \frac{dc^*}{dx} \right|, \quad (3)$$

where the gradient of the light speed c^* in the fiber is taken at the emulated black hole horizon. In the laboratory reference frame the Hawking quanta will have frequencies in the $(\omega_0, \omega_0 + \Delta\omega)$ range, where

$$\Delta\omega = \frac{1}{2\pi} \left| \frac{dc^*}{dx} \right| \quad (4)$$

Because of the anomalous dispersion, these quanta can escape the horizon (their speed is faster than the speed

of the optical pulse). Using eq.(2) we can re-write eq.(4) as

$$\Delta\omega = \frac{c}{2\pi n^2} \left| \frac{dn}{dx} \right| = \frac{1}{2\pi n} \left| \frac{dn}{dt} \right| = \frac{n_2}{2\pi n} \left| \frac{dI}{dt} \right|, \quad (5)$$

where we have replaced the refractive index gradient by the time derivative of the refractive index (this is a valid replacement since similar to ref.⁴ the local speed of light variations due to the propagation of the pulse through the waveguide depend on $(x + c^*t)$). The final result in eq.5 appears to be identical to the well-known effect of spectral broadening of an optical pulse due to self-phase modulation. Indeed, the effect of self-phase modulation is usually derived by considering local phase variations across the optical pulse due to intensity dependent refractive index described by eq.(2). The local shift in frequency $\Delta\omega(x)$ in the given location x in the pulse in the reference frame comoving with the pulse is obtained as

$$\Delta\omega(x) = \frac{d\phi(x)}{dt} = \frac{d}{dt}(\omega_0 t - kx) = -\frac{2\pi x}{\lambda_0} \frac{dn}{dt} = \frac{2\pi x n_2}{\lambda_0} \left(\frac{dI}{dt} \right), \quad (6)$$

where λ_0 is the wavelength of light in free space. Thus, the spectral width of the pulse is broadened near the leading and the trailing edges of the pulse where dI/dt is nonzero. The spectral broadening of the pulse appears to be the same as the one (eq.5) obtained from the estimate based on the expression for the effective Hawking temperature derived in ref.⁴.

The effect of self-phase modulation was seen in numerous experiments. The more striking examples of this effect are described in refs.10,11 in which the effective Hawking temperature is very large (see ref.⁷) and a broad so-called white light supercontinuum is generated while intense femtosecond optical pulses propagate through optical fibers exhibiting anomalous dispersion in a wide spectral range. Thus, we may conclude that the Hawking radiation has been already observed in numerous fiber optics experiments.

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FIG. 1: Light intensity and the speed of light in an optical fiber in the reference frame comoving with the intense short optical pulse. Low frequency photons cannot escape the leading edge of the pulse, which constitutes an effective black hole horizon for low frequencies. This picture is similar to Fig.2 from ref.⁸.

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